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VIBRATIONS OF A FAN JET IN RUNNING A SUPERSONIC JET INTO A NOTCHED OBSTACLE

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Some kinds of vibrations of a fan jet are described. A physical model of one kind of self-excited vibrations of the fan jet is formulated. The formulas to calculate the frequencies of self-excited vibrations are given.

The interaction between a supersonic jet and an obstacle is an important factor in a number of technical devices. The aerodynamic emitters of a sound (whistles) and the system consisting of an exhaust rocket jet and a launching device which interact with each other are widely known. In these cases, the nonstationary phenomena in the flow are of determining value. Pulsations which occur in running an axisymmetrical supersonic jet into a flat obstacle (Hartmann effect) and an open end of a plugged tube (Hartmann–Sprenger effect) are studied quite adequately. Here attention is mainly paid to the nonstationary processes occurring in the incident jet and the tube. The vibrations in the jet spreading over the obstacle or the tube flange are not yet properly explained. Known are periodic vibrations of a fan jet which are connected with vibrations in the incident jet or tube [1]. However, the periodic pressure pulsations in the tube and the fan jet do not correlate under some conditions (there is no constant phase shift). Therefore, one can study the fan jet as a separate object of research.

It is known that with variation in the parameters that determine a jet flow, there exist zones of parameters in which intense irregular and periodic vibrations are observed. In these zones, the mechanisms of periodic vibrations differ, i.e., the vibrations are described by different physical models. This study describes the varieties of vibrations of a fan jet with indication of a set of stationary parameters that determines the existence zone of vibrations of a given type. We assume that the stationary flow structure is specified as accepted in studying the nonstationary phenomena in the jets.

Figure 1 shows the flow pattern of a supersonic underexpanded axisymmetrical jet 7 running into a flat obstacle 3 with a notch 2. The obstacle is located in the first barrel of the jet normal to the jet axes. The conditions for vibrations appear when the following parameters of the system are changed: the noncalculation of the incident jet n, the nozzle–obstacle distance x_0, and the diameter d and depth h of the notch. These vibrations can be both periodic and nonperiodic. In the case of periodic vibrations, a fan jet 4 either executes forced vibrations with a discrete frequency or enters, as a unit, into the self-vibrating system. Here the mechanisms of self-vibrations can be different.

Glaznev and Demin [1] showed that in definite off-design modes of an incident jet 7 and at definite distances from the nozzle cut to the flat obstacle there is a self-excited vibration mode with emission to the space of sound waves. Here the fan jet is an emitter of sound. The feedback in this self-oscillating system is closed by sound waves through the external space between the nozzle cut 1 and the boundary of the fan jet 4. The vibration frequency f is mainly determined by the size x_0 and the sound velocity a. The frequency can be calculated by the empirical formula [2] $f = a(k + 1.25)/(2x_0)$, where $k = 0, 1, 2, \ldots$. In this case, the fan jet is an active element of the self-oscillating system. If there is a cylindrical notch 2 coaxial to the incident jet

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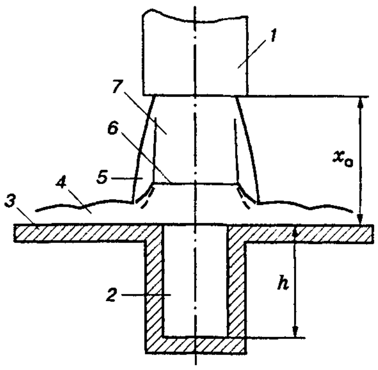

Fig. 1

7 in the obstacle (Fig. 1) and the notch diameter is smaller than the diameter of a direct shock 6 before the obstacle, the notch affects little the self-excited vibrations. This is true for quite significant noncalculations of the incident jet ($n > 1.5$). For these values of the ratio between the pressures at the nozzle cut and in the ambient medium in the free (no obstacle) jet 7, a direct shock (the Mach disk) whose diameter is comparable with the jet diameter is formed.

If the edge of the notch 2 is in the region of deceleration of a compressed layer 5 in a section where the static pressure rises along the radius from the axis of symmetry to the periphery for these values of n, self-excited vibrations connected with the presence of a notch in the obstacle occur. The mechanism of these self-excited vibrations was considered in [3] for the case where the depth of the notch is greater than its diameter. The vibration frequency $f = 0.25a/L$ is close to the eigenfrequency of the tube of length L closed at one end. Here L is the distance from the notch bottom to the direct shock in front of the obstacle. During intense vibrations, owing to the nonlinear character of the process, the frequency can be smaller than the expected one (the discrepancy can reach 20%). In this case, the fan jet 4 executes forced vibrations under the action of self-excited vibrations in the notch. At the moment when the outflow from the notch occurs during vibrations, the fan jet can detach from the obstacle.

We found experimentally that in the case where the notch diameter is smaller than the diameter of an incident jet, for a noncalculation of the jet close to unity, self-excited vibrations, whose frequency is mainly determined by the distance between the direct shock and the obstacle surface behind the shock, occur. These vibrations arise if the direct shock is located in the second half of the barrel in the jet, where the Mach number decreases along the jet axis. In this case, its presence influences considerably the vibration frequency, irrespective of the notch diameter, because the frequency is determined by the distance from the shock to the notch bottom. The fact that the occurrence of self-excited vibrations is influenced by the distribution of the Mach number along the jet axis and is not affected by the notch diameter indicates that, for a small noncalculation of n, a mechanism of vibrations different from that for $n > 1.5$ is realized. The standard model of this phenomenon is not available.

The authors [4] described some kinds of vibrations of a fan jet when the diameter of a notch is close to or exceeds the diameter of the jet incident on the obstacle. In this case, the fan jet is separated from the obstacle and can vibrate owing to ejection of the fan jet. Figure 2 shows the pattern of this flow (the flow regimes are different on the right and on the left). As a result of ejection, the pressure between a fan jet 3 and the plane of an obstacle 8 drops. Under the action of the pressure difference along the normal to the axis of the fan jet the latter bends toward the obstacle. If the obstacle has a large ("infinite") diameter, the fan jet "adheres" to the obstacle and begins to flow along it. Here the pressure in a "bubble" 4 between the fan jet and the obstacle is smaller than atmospheric pressure. The fan jet can bend around the obstacle when the obstacle diameter is smaller. As is shown on the right of Fig. 2, in this case, the gas flows as a circular jet 5 in the same direction as a jet 2.

If a fan jet 3 flows from the notch at an angle less than 45° to the obstacle and the attachment point

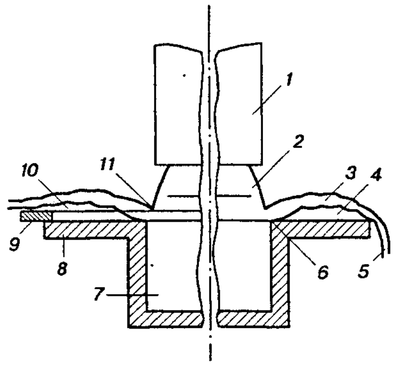

Fig. 2

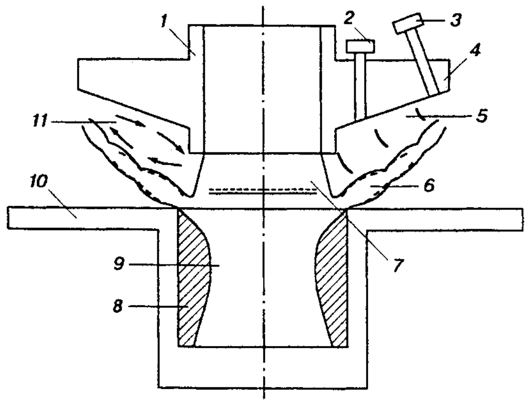

Fig. 3

to a large-diameter obstacle is on a site of the fan jet where there is a barrel, the fan jet vibrates irregularly. In [4], this was explained by the fact that shock waves are generated when the supersonic flow touches the wall. In addition, the static pressure increases downstream in the second half of the barrel. Therefore, in the field of attachment, the distribution of the static pressure appears along the wall which leads to separation of the boundary layer. The flow detaches. The fan jet is thrown away from the obstacle, but the force created by ejection again returns the fan jet to the obstacle. If, in the case shown on the right of Fig. 2, the diameter of an obstacle 8, is reduced, it is clear that the difference between the pressures on both sides of a fan jet begins to decrease. For a definite diameter of the obstacle, the fan jet 3 becomes "bell-shaped", as is shown in Fig. 3. In this case, owing to ejection, the pressure in the region 5 is smaller than the pressure in the space between the boundary of a jet 6 and an obstacle 10. In the interval of variation in the obstacle diameter, there is a transition zone, in which the bell-shaped fan jet becomes spontaneously a "bubble"-shaped fan jet. Here the obstacle undergoes irregular shock loads, and the direction of the fan jet changes. According to the flow regime, the fan jet can be in one of the stationary positions for a large time lapse, making short jumps-over to another position. There are regimes in which the periods during which the jet 6 is in these positions are approximately equal and correspond to a frequency of approximately 400 Hz. However, the stable-phase self-excited vibrations were not observed. The parameters of the device are indicated below. Since the pressure pulsations in a notch 9 did not always correlate with those at the obstacle under the fan jet, one can assume that the "jumps-over" of the fan jet are caused by radial vibrations of the boundaries of the incident jet. To convince oneself that there is a correlation between the position of the boundary of the incident jet and the dimension of the fan jet at a constant total pressure of the nozzle 1, the obstacles were placed in space 5 (Fig. 3). The latter hindered the ejection from space 5, so that the pressure in it arose. As a result, the diameter of the incident jet decreased and, hence, the size of the slit from which the fan jet 6 flowed out increased. An increase in the dimension of the first barrel of the fan jet was observed and, consequently, the long range of the fan jet and its ejection capacity increased. The configuration of the fan jet also depends on these properties.

The authors attempted to make the "jumps-over" of the fan jet regular, i.e., to generate self-excited vibrations. For this purpose, the adjustment of the pressure in the prechamber of a nozzle 1 (see Fig. 2) allowed one to select a flow regime such that the fan jet was for the most part in the "bubble" position. A "ledge" was placed at the attachment point of the fan jet to the obstacle. To do this, a ring 9 was fastened to an obstacle 8 (Fig. 2). It was expected that at the moment of attachment to the obstacle, the fan jet is decelerated by the "ledge," a return flow occurs along the obstacle, the pressure in the "bubble" increases, and the fan jet becomes a "bell-like" jet. The effect of ejection returns the jet to the obstacle, and self-excited vibrations arise. However, the presence of the "ledge" only changed the shape of the "bubble" and shifted the attachment point. The fan jet did not separate from the obstacle. The variation in the dimensions and the

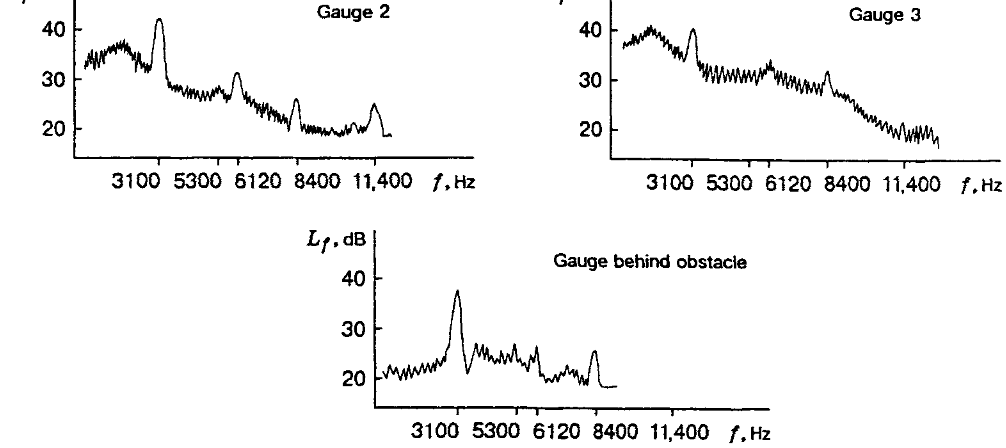

Fig. 4

site of the "ledge" did not lead to separation either.

With increase in the diameter of the flange 4 of the nozzle 1 (see Fig. 3), being in the "bell-like" position, a fan jet 6 can run into the flange. Spreading over the flange, a portion of the gas initiates a return flow 11, which increases the pressure in space 5. As stated above, this causes an increase in the dimensions of the barrels of the fan jet. Just as in the case with a "ledge," the fan jet takes a position which corresponds to the inflow-to-outflow balance of the gas in space 5. Self-excited vibrations are not generated in this case.

However, if the fan jet passes along the edge of the flange 4, it begins to vibrate. Here, sound waves of a definite tone are generated. In the model shown in Fig. 3, regular vibrations were observed on an interval of variation in the total pressure in the prechamber $P = (3.25\text{–}4.25) \cdot 10^5$ Pa when the distance between the nozzle cut and the obstacle was $x_0 = 8$ mm. In this case, a supersonic jet 7 outflows from a nozzle 1 for the Mach number $M_a = 1$. The nozzle diameter is $d_a = 16$ mm. The nozzle wall is 2 mm thick. The nozzle has a flange 4 of diameter 53 mm. The thickness of the flange near the nozzle is 10 mm, and the thickness at the edge of the flange is 4 mm. The nozzle cut and the edge of the flange are 3.5 mm apart. The diameter of the obstacle 10 is 74 mm. The diameter of the notch 9 at the exit is 24 mm, and its depth is 20 mm. An insertion 8 sets a 45° angle of efflux of the fan jet 6 between the jet axis and the plane of the obstacle 10. Two gauges 2 and 3 are built in the flange 4 to measure the pressure and pressure pulsations in space 5. The receiving orifice of these gauges is 0.5 mm in diameter. The phase characteristics of the gauges were ignored. The control gauge of pressure pulsations was positioned in the acoustic field behind the obstacle. The vibration phases of the shadow pattern of the fan jet were photographed using a stroboscopic method. The stroboscope was excited by the signal from the gauge 2. The spectrograms of the signals from three gauges were recorded at a pressure of $P = 3.57 \cdot 10^5$ Pa in the prechamber. One can see the peak of the level of pulsations L_f, which corresponds to the tone radiation with frequency $f = 3100$ Hz and the small peaks, which corresponds to frequencies of 5370, 6120, and 8420 Hz, on all the spectrograms (Fig. 4). The tone signal of the gauge 3 is extremely noisy. The signal from the gauges 2 has a well expressed regular character in an acoustic field as well. A comparison of the signals generated by the gauges 2 and 3 passed through a narrow-band (6%) filter at the frequency $f = 3100$ Hz shows that the pressure pulsation registered by the gauge 2 lags behind in phase the pressure pulsation at the site where the gauge 3 is installed at 41.5°. The gauge 3 shows an average pressure greater than the atmospheric pressure, and the gauge 2 registers a pressure smaller than atmospheric pressure. This means that during vibrations the fan jet arrives at the receiving orifice of the gauge 3. As the stroboscopic photographs show, the angle between the axis of the first barrel of the fan jet 6 (see Fig. 3) and the obstacle surface during pulsations remains almost unchanged. Here the length and thickness of the first barrel pulse in phase. The vibrations of the fan jet are symmetric relative to the axis of symmetry common with the jet 7. Figure 3, which is drawn on the basis of photographs, shows two extreme positions of the fan jet. The curves refer to the boundaries of the barrels of the fan jet, which are seen in the photographs, obtained by a direct shadow method. The dotted curves refer to the position when the maximum pressure occurs at the edge of the flange 4. The solid curve refers to the position of the fan jet when the pressure is minimum at the edge of this flange. It is worth noting that the radial displacements of the fan jet relative to the common axis of symmetry occur mainly in the tail part of the fan jet near the flange edge. In one of the experiments, the cavity of the notch 9 was half-filled with cotton wool, which did not exert a noticeable effect on the vibrations. Therefore, the cavity of the notch does not maintain self-excited vibrations.

Based on the results, we propose the following mechanism of self-excited vibrations. Owing to ejection, the pressure in space 5 (see Fig. 3) bounded by the nozzle flange and the boundary of the incident and fan jets is smaller than atmospheric pressure. The fan jet touches the edge of the flange 4. The total jet head is partially restored at the edge. This is responsible foe the higher pressure at the edge and the eddy motion in space 5.

We assume that, owing to transverse perturbations, the fan jet is displaced from the flange edge. The total restored pressure decreases, and the pressure at the edge of the flange falls off. A rarefaction wave runs toward the incident jet along the flange. When the wave reaches the boundary 7 of the incident jet, the diameter of the latter increases, because the noncalculation of the jet increases. The width of the ring between the notch wall and the boundary of the incident jet becomes smaller. The fan jet becomes thinner, and its first barrel shorter. The range of this jet becomes shorter and it is further curved. The fan jet is restructured downstream. The perturbation velocity is equal to the sound velocity plus the flow velocity. The fan jet occupies the position indicated by the dotted curve in Fig. 3. Because the jet begins to run into the flange edge, the pressure at the edge increases. A pressure wave runs into space 5. When it reaches the boundary 7 of the incident jet, the latter decreases in the diameter. This will give rise to an increase in the thickness and length of the first barrel of the fan jet, and, hence, the long-range of the entire jet, which takes the position indicated by the solid curve in Fig. 3. Since the fan jet passes by the flange in this position, the pressure at the edge decreases. A rarefaction wave propagates to space 5, and the process is repeated. Thus, the perturbation wave should pass twice the distance from the flange edge to the boundary of the incident jet and back for a period of vibrations. The length of this path and the perturbation velocity of the distribution determine the frequency of self-excited vibrations. For simplicity, one can separate a "monopole" type source of sound in this self-oscillating system. It is located at the flange edge and generates sound waves, which are shown by three arcs in Fig. 3, toward the boundary of the incident jet. Characteristic of the flow regime considered is the fact that a small change in the static pressure at the boundary 7 of the incident jet (in space 5) causes a more drastic change in the pressure at the edge of the flange 4. In the limiting cases, the fan jet either restores the total head or does not reach the edge. Therefore, the boundary of the incident jet with the fan jet and the flange edge can be regarded as an "amplifier" of pressure perturbations. In this case, one can state that the sound waves propagating from the flange edge to the boundary of the incident jet realize the "feedback" in this self-vibrating system. Vibrations are excited if the losses in the "feedback" circuit are compensated by the increment in the "amplifier."

This model of self-excited vibrations allows us to derive a formula to estimate the frequency. The phase shifts between the signals of the pressure gauges 3 and 2 (see Fig. 3) is $\Delta\varphi = 41.5°$. For a frequency $f = 3100$ Hz and a distance $\Delta x = 11.2$ mm between the gauges, this shift corresponds to the phase velocity a of the pressure perturbation along the flange ($a = 360 f\Delta x/\Delta\varphi = 300$ m/sec). This velocity is close, within the experimental error, to the sound velocity in the atmosphere. The average flow velocity along the axis of the fan jet is taken to be equal to 1.5 sound velocities. It follows from further calculation that the large error in the determination of the average velocity (by $0.5a$) changes the calculated frequency by no more than 6%. The phase perturbation velocity in this fan jet is $v = 2.5a$. As indicated above, the pressure at the flange edge is determined by the transverse displacement of the fan jet. This displacement is set by the displacement of the incident-jet boundary, and the latter is displaced under the action of the pressure wave. Here the vibrations of the boundary of the incident jet are behind, in phase, the pressure oscillations at the boundary by 90°. Indeed, the displacements of the boundary are equal to the displacement of the particles in the wave. If the acoustic wave near the incident jet is considered flat, the pressure and the velocity in the wave change in phase. Let the pressure P and the velocity v change according to the law $v = cP = cP_1 \cos(\omega t)$. The displacement of the boundary x is the time integral of the velocity:

$$x = cP_1 \int \cos(\omega t)\, dt = cP_1 \sin(\omega t)/\omega.$$

However, the function $\sin(\omega t)$ is shifted in phase relative to $\cos(\omega t)$ by 90°. In the case considered, this is equivalent to the delay of the wave by an additional period equal to 90/360 of the total period. If the distance l between the flange edge and the boundary of the incident jet is approximately equal to the length of the fan jet, the period of vibrations can be estimated from the condition that the perturbation passes twice the entire phase path for a time that is a multiple of the period of vibrations T:

$$2\left(\frac{l}{a} + \frac{l}{2.5a} + 90\frac{T}{360}\right) = T(k+1) \quad (k = 0, 1, 2, \ldots).$$

With allowance for the fact that $f = 1/T$, after transformations we obtain the desired formula: $f = (a/l)(2k+1)/5.6$.

If it is assumed that $a = 340$ m/sec and $l = 20 \cdot 10^{-3}$ m, we have $f = 3040$ Hz for $k = 0$ and $f = 9100$ Hz for $k = 1$. Therefore, the measured frequencies, which are 3100 and 8420 Hz, should be assigned to the "modes," i.e., to the independent vibrations occurring in the vibrating system. Only the two frequencies were registered by a gauge in the acoustic field behind the obstacle. A frequency of 6120 Hz is a "harmonic" frequency. It appears in the spectrogram owing to the expansion of the nonsinusoidal signals with a period of 1/3100 sec obtained from the gauges 2 and 3 (see Fig. 3) into a Fourier series. The two weak peaks with frequencies of 5370 and 11,400 Hz in the spectrograms of the gauges 2 and 3 are, apparently, the result of the nonlinear interaction between the above "modes," because the frequencies of the weak peaks are equal to the difference and sum of these "modes."

It is noteworthy that, according to the suggested model, the acoustic waves generated to the external space by the fan jet do not participate in maintaining the self-excited vibrations. This is one of the differences between the self-excited vibrations considered and those described in [1].

A comparison of the flow patterns depicted on the left of Figs. 3 and 2 allows one to conclude that Fig. 2 contains the same elements of the self-vibrating system as those in Fig. 3. A "monopole"-type emitter of a sound is on the internal edge of the ring 9 (Fig. 2). The sound emitted by it can influence the boundary of the incident jet 2 similarly to the case given in Fig. 3. Figure 2 shows that self-excited vibrations do not arise only because the losses of the vibration energy in the "feedback" are not compensated by "the amplifier." The latter is probably connected with the fact that "the monopole" in Fig. 2 emits a sound mainly to space 10 between the fan jet and the obstacle. However, the sound waves on this site cannot displace the separation point 6 of the jet from the obstacle. These waves do not displace the point 11 as well, because they are shielded by the supersonic flow in the fan jet. Therefore, these waves cannot change significantly the width of the slit and, hence, the dimensions of the first barrel of the fan jet. Thus, the "feedback" in Fig. 2 is weaker than that in the flow shown in Fig. 3.

We note that when the obstacle is displaced from the first barrel of the incident jet to the second or successive barrels, the total head of the fan jet decreases and this variety of intense vibrations of the fan jet is not observed. Here we did not consider the cases where the diameter of the deep notch is so much larger than the diameter of the jet that the flow should be regarded as a flow incoming in a closed cavity, rather than a flow running into an obstacle. Under these conditions, intense vibrations of other types are possible.

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